A Modified Noncoherent PN Code Acquisition Scheme

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***Abstract*— A modified non-coherent detector for PN code acquisition of direct sequence spread spectrum signals is introduced. The proposed detector, referred as Differential Non-Coherent(DNC) detector, is suitable for popular CDMA systems and it performs better than classical non-coherent detector. The performance improvement is shown to be 2dB in terms of SNR while false alarm and detection probabilities are used as measurement criteria. Under a good PN code sequence assumption, an exact analysis of the DNC detector is derived. The Receiver Operating Characteristics (ROC) results for DNC detector are then plotted against non-coherent detector results to show performance improvement. Also, a system level simulation model of the proposed detection scheme is developed using a popular system level design tool and the simulated performance results are compared against theoretical results.**

***Keywords*- Acquisition,CDMA,noncoherent,detection**

I. Introduction

Many of the present day communication systems, especially the wireless ones, use spread spectrum technology for various reasons, such as interference immunity, higher ranging and multiple access support. Accurate synchronization plays a cardinal role in all these spread spectrum systems. Typically, the process of synchronization between the incoming spreading PN code and the local PN code is done in two steps. First step is the code acquisition and this is followed by tracking. Our focus here will be on PN Code Acquisition. PN code acquisition problem has been extensively studied and discussed in the literature, for example [1] and [2].

Since the PN code acquisition happens before perfect carrier synchronization, the Direct Sequence Spread Spectrum (DSSS) waveform is acquired typically through non-coherent detection. A non coherent detector comprise of bandpass filter, PN matched filter and square law detector [1].

In this paper, we propose a modified non-coherent detector for PN code acquisition which we refer as *differential non-coherent* (DNC) detector. This detector provides better performance in terms of detection and false alarm probabilities compared to classic non-coherent(NC) detector[1]. This is achieved at the expense of small increase in structural complexity.

The paper is organized as follows. Section II introduces the system model and structure of the proposed detector. In Section III acquisition performance in terms of detection and false alarm probabilities, is derived. Section IV discusses the performance results of the new detector and compares them with that of non-coherent detector. Simulation results are provided in Section V. Section VI concludes the paper.

II. System Model

The receiver structure for a DS/BPSK using differential non-coherent (DNC) detector is shown in Fig.(1). The dotted box section essentially represents the detector. In the presence of additive white Gaussian noise it is well known that optimum receiver performs matched filtering operations [1]. These include down conversion, matched filtering with the complex conjugate of the transmit filter $H^*(f)$, and sampling at the chip intervals. This is followed by active PN correlator (or chip matched filter). The correlated output is then passed through the detector. The DNC detector consists of two delay lines, followed by complex multiplier and a squaring unit. Each sample output from the detector Z_k is compared with a given threshold. Depending upon the comparator output, the receiver either accepts the corresponding PN code phase and goes into tracking mode or continues to search. For the system under discussion, no post integration is considered.

For the DSSS signal transmission, N is the spread factor and each information bit is spread with N chips. At the receiver, the PN code is acquired from this sequence of spread information bits.

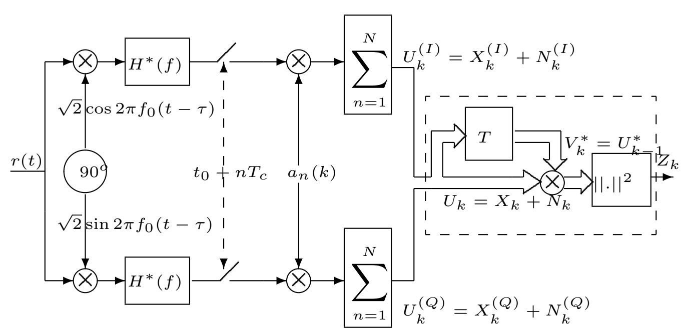


Fig. 1. Modified Non coherent detector

The detector structure of traditional non-coherent receiver is shown in Fig.2(a) along with the differential non-coherent detector in Fig.2(b). In Fig.(2) the entire detection operation is shown in complex baseband equivalent form. Also, chip matched filter is used in the place of active PN correlator.

III. DNC Detector Analysis

The complex received signal at the front end of the acquisition receiver in Fig.(1) is

$$r(t)=\sqrt{2E_c(k)}\sum_n x_n(k)a_n(k)h(t-nT_c)\cos(2\pi f_0 t+\Phi_k)+N(t) \quad (1)$$

where $a_n(k)$ is the PN code and $x_n(k)$ is the data sequence synchronized to the PN sequence. Both $x_n(k)$ and $a_n(k)$ are in duo binary form (±1). f_0 is the carrier frequency and Φ_k accounts for the phase. E_c is the received chip energy and $N(t)$ is a complex white Gaussian process with two sided power spectral density $N_0/2$. The output sample after the correlator and de-spreading stage can be written as [1]

$$\begin{aligned} U_k^{(I)} &= X_k^{(I)} + N_k^{(I)} \\ U_k^{(Q)} &= X_k^{(Q)} + N_k^{(Q)} \end{aligned}$$

where $N_k^{(I)}$ and $N_k^{(Q)}$ are zero mean real Gaussian random variables.

The detector stage equations can now be expressed as,

$$\begin{aligned} U_k &= U_k^{(I)} + jU_k^{(Q)} \\ V_k &= U_{k-1} \\ Y_k &= U_k V_k^* \\ &= U_k U_{k-1}^* \\ Z_k &= ||Y_k||^2 \\ &= ||U_k||^2 ||U_{k-1}||^2 \end{aligned} \quad (2)$$

A. False alarm probability

Under zero Hypothesis H_0, U_k is equal to the complex Gaussian random process itself. We can then proceed as follows.

$$\begin{aligned} U_k &= N_k^{(I)} + jN_k^{(Q)} \\ ||U_k||^2 &= \left(N_k^{(I)}\right)^2 + \left(N_k^{(Q)}\right)^2 \\ ||U_{k-1}||^2 &= \left(N_{k-1}^{(I)}\right)^2 + \left(N_{k-1}^{(Q)}\right)^2 \end{aligned} \quad (3)$$

From Eq.(2) and Eq.(3), the detector output under H_0 can be written as

$$Z_k = \Psi\Phi \quad (4)$$

where

$$\begin{aligned} \Psi &= \left(N_k^{(I)}\right)^2 + \left(N_k^{(Q)}\right)^2 \\ \Phi &= \left(N_{k-1}^{(I)}\right)^2 + \left(N_{k-1}^{(Q)}\right)^2 \end{aligned}$$

It is well known [3],[4] that sum of squares of two zero mean Gaussian random variables is exponential. We can thus write the probability distribution functions of Ψ and Φ as

$$\begin{aligned} f_\Psi(\psi) &= \frac{1}{2\sigma^2}\exp\left(\frac{-\psi}{2\sigma^2}\right) \quad \psi \geq 0 \\ f_\Phi(\phi) &= \frac{1}{2\sigma^2}\exp\left(\frac{-\phi}{2\sigma^2}\right) \quad \phi \geq 0 \\ f_{\Psi\Phi}(\psi,\phi) &= \frac{1}{4\sigma^4}\exp\left(\frac{-(\psi+\phi)}{2\sigma^2}\right) \end{aligned} \quad (5)$$

Given the joint probability distribution function $f_{\Lambda\Theta}(\lambda,\theta)$ of two random variables Λ and Θ, the pdf of their product $\Gamma = \Lambda\Theta$ is [3] given by

$$f_\Gamma(\gamma) = \int_{-\infty}^{\infty} \frac{1}{|\omega|} f_{\Lambda\Theta}\left(\omega, \frac{\gamma}{\omega}\right) d\omega \quad (6)$$

When the random variables Λ and Θ are independent Eq.(7) can be further simplified in terms of their respective density functions as

$$f_\Gamma(\gamma) = \int_{-\infty}^{\infty} f_\Lambda(\omega) f_\Theta\left(\frac{\gamma}{\omega}\right)\frac{1}{|\omega|} d\omega \quad (7)$$

Using Eq.(7) on Eq.(5) we can write the pdf of the detector output Z represented by $f_Z^{(0)}(z)$ as,

$$\begin{aligned} f_Z^{(0)}(z) &= \int_{-\infty}^{\infty} \frac{1}{|\omega|} f_{\Psi\Phi}\left(\omega, \frac{z}{\omega}\right) d\omega \\ &= \frac{1}{4\sigma^4}\int_0^\infty \frac{1}{|\omega|}\exp\left[-\frac{\left(\omega + \frac{z}{\omega}\right)}{2\sigma^2}\right] d\omega \\ &= \frac{1}{4\sigma^4}\int_0^\infty \frac{1}{\omega}\exp\left[-\frac{\left(\omega + \frac{z}{\omega}\right)}{2\sigma^2}\right] d\omega \end{aligned} \quad (8)$$

From [5] PP-385,3.471,12 we have

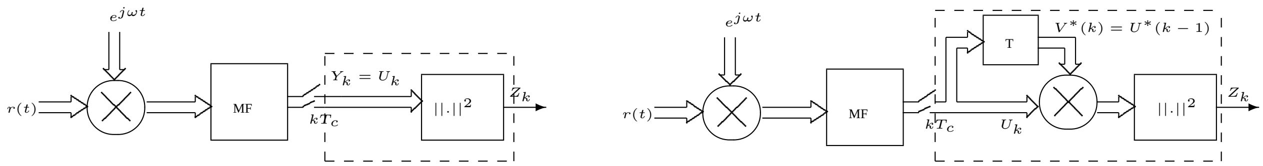
$$\int_0^\infty x^{\nu-1}\exp\left(-x - \frac{\mu^2}{4x}\right)dx = 2\left(\frac{\mu}{2}\right)^\nu K_{-\nu}(\mu) \quad (9)$$

As a special case of Eq.(9) when $\nu = 0$ we have the finite integral in closed form as

$$\int_0^\infty \frac{1}{x}\exp\left(-x - \frac{\mu^2}{4x}\right)dx = 2K_0(\mu) \quad (10)$$

Using Eq.(10) on Eq.(8) we can simplify $f_Z^{(0)}(z)$ further through the following steps.

$$\begin{aligned} f_Z^{(0)}(z) &= \frac{1}{4\sigma^4}\int_0^\infty \frac{1}{|\omega|}\exp\left[-\frac{\left(\omega + \frac{z}{\omega}\right)}{2\sigma^2}\right] d\omega \\ &= \frac{1}{4\sigma^4}\int_0^\infty \frac{1}{|\omega|}\exp\left[-\frac{\left(\omega + \frac{\theta^2}{\omega}\right)}{2\sigma^2}\right] d\omega \ \leftarrow z = \theta^2 \\ &= \frac{1}{4\sigma^4}\int_0^\infty \frac{1}{\omega}\exp\left(-\frac{\omega}{2\sigma^2} - \frac{\theta^2}{2\omega\sigma^2}\right) d\omega \\ &= \frac{1}{4\sigma^4}\int_0^\infty \frac{1}{2\sigma^2 x} e^{\left(-x - \frac{\theta^2}{4\sigma^4 x}\right)} 2\sigma^2 dx \ \leftarrow x = \frac{\omega}{2\sigma^2} \\ &= \frac{1}{4\sigma^4}\int_0^\infty \frac{1}{x}\exp\left(-x - \frac{\mu^2}{4x}\right) dx \ \leftarrow \mu = \frac{\theta}{\sigma^2} \\ &= \frac{1}{4\sigma^4} 2K_0(\mu) \end{aligned}$$



(a) conventional non-coherent detector

(b) modified non-coherent detector

Fig. 2. Detector structure

$$= \frac{1}{2\sigma^4} K_0\left(\frac{\sqrt{z}}{\sigma^2}\right) \tag{11}$$

where $K_0(t)$ is the modified Bessel function of the second kind of order 0.

From the likelihood function in Eq.(11) we can compute the false alarm probability

$$\begin{aligned} P_F &= \int_{V_T}^{\infty} f_Z^{(0)}(z)dz \\ &= \frac{1}{2\sigma^4}\int_{V_T}^{\infty} K_0\left(\frac{\sqrt{z}}{\sigma^2}\right)dz \\ &= \int_{\frac{\sqrt{V_T}}{\sigma^2}}^{\infty} uK_0(u)du \quad \leftarrow u = \frac{\sqrt{z}}{\sigma^2} \\ &= \frac{\sqrt{V_T}}{\sigma^2} K_1\left(\frac{\sqrt{V_T}}{\sigma^2}\right) \end{aligned} \tag{12}$$

where $K_1(t)$ is the modified Bessel function of the second kind of order 1, and V_T is the threshold. In the final stage of Eq.(12) we have used the identity Eq.(13)

$$\int xK_0(x)dx = -xK_1(x) \tag{13}$$

B. Probability of Detection P_D

For the hypothesis H_1 we have

$$\begin{aligned} Z_k &= |Y_k|^2 \\ &= |U_k|^2|V_k|^2 \\ &= \Lambda\Delta \leftarrow \Lambda = |U_k|^2, \Delta = |V_k|^2 \end{aligned}$$

where

$$\begin{aligned} \Lambda &= \left(X_k^{(I)} + N_k^{(I)}\right)^2 + \left(X_k^{(Q)} + N_k^{(Q)}\right)^2 \\ \Delta &= \left(X_{k-1}^{(I)} + N_{k-1}^{(I)}\right)^2 + \left(X_{k-1}^{(Q)} + N_{k-1}^{(Q)}\right)^2 \end{aligned}$$

Since Λ and Δ are sum of squares of two non zero mean Gaussian random variables of arbitrary but equal variances, the probability density function of them will be the well known *scaled non central* χ^2. We can write the *pdf*s as

$$f_\Lambda(\lambda) = \left\{ \begin{array}{ll} \frac{1}{2\sigma^2} e^{-\left(\frac{\lambda+\mu^2}{2\sigma^2}\right)} I_0\left(\sqrt{\frac{\lambda\mu^2}{\sigma^2}}\right), & \lambda \geq 0 \\ 0, & \text{elsewhere} \end{array} \right\} \tag{14}$$

and

$$f_\Delta(\delta) = \left\{ \begin{array}{ll} \frac{1}{2\sigma^2} e^{-\left(\frac{\delta+\mu^2}{2\sigma^2}\right)} I_0\left(\sqrt{\frac{\delta\mu^2}{\sigma^2}}\right), & \delta \geq 0 \\ 0, & \text{elsewhere} \end{array} \right\} \tag{15}$$

where we denoted the signal power $||x||^2$ by the symbol μ^2.

Using Eq.(14) and Eq.(15) on Eq.(7) we get

$$f_Z^{(1)}(z) = \int_0^{\infty} \frac{e^{-\frac{\mu^2+\omega}{\sigma^2}} I_0\left(\frac{\mu\sqrt{\omega}}{\sigma^2}\right) e^{-\frac{\mu^2+\frac{z}{\omega}}{\sigma^2}} I_0\left(\frac{\mu\sqrt{\frac{z}{\omega}}}{\sigma^2}\right)}{4\sigma^4\omega} d\omega \tag{16}$$

The detection probability can be worked out into a numerically computable form as follows.

$$P_D = \int_{\beta}^{\infty} f_Z^{(1)}(z)dz \tag{17}$$

On substitution of Eq.(16) on Eq.(17), followed by some algebraic manipulations we arrive at the following finite integral expression for the detection probability.

$$P_D = \frac{1}{2\sigma^2}\int_0^{\infty} e^{-\left(\frac{\omega+\mu^2}{2\sigma^2}\right)} Q\left(\frac{\mu}{\sigma}, \sqrt{\frac{\beta}{\omega\sigma^2}}\right) I_0\left(\frac{\mu\sqrt{\omega}}{\sigma^2}\right) d\omega \tag{18}$$

where $Q(\alpha, \beta)$ is the first order Marcum-Q function. The generalized Marcum-Q function of order M is defined as

$$Q_M(\alpha, \beta) = \frac{1}{\alpha^{M-1}}\int_{\beta}^{\infty} x^M e^{-\left(\frac{x^2+\alpha^2}{2}\right)} I_{M-1}(\alpha x)dx \tag{19}$$

IV. Results and Discussion

The analytical expressions of false-alarm and detection probabilities of DNC detector are numerically evaluated. The P_F and P_D are computed for various values of SNR per bit (γ_b) where $\gamma_b = \mu^2/2\sigma^2$. The Fig.(3) shows ROCs (i.e. P_D vs P_F) of DNC and NC detectors with γ_b=5dB and 10dB. The results for NC detector are analyzed in [1]. The curves show that the DNC provides superior performance over NC.

The Fig.(4) shows the DNC-ROC with γ_b=3dB and NC-ROC with γ_b=5dB. These two curves match very closely showing that DNC has got 2dB advantage over NC. Also shown in the figure are DNC-ROC with γ_b=8dB and NC-ROC with γ_b=10dB which again match closely.

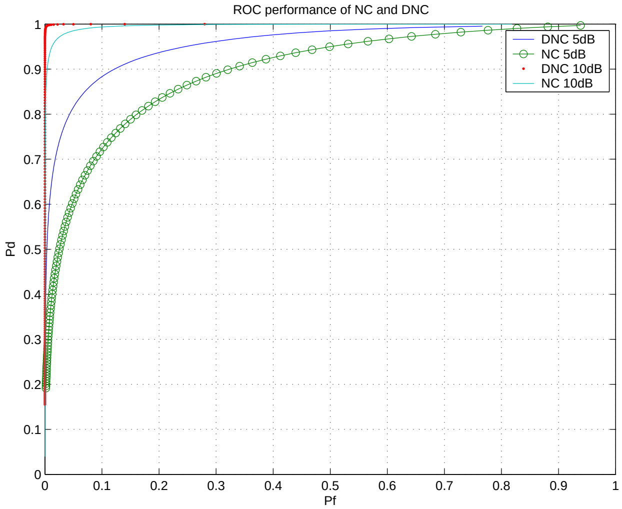


Fig. 3. ROC performance comparison

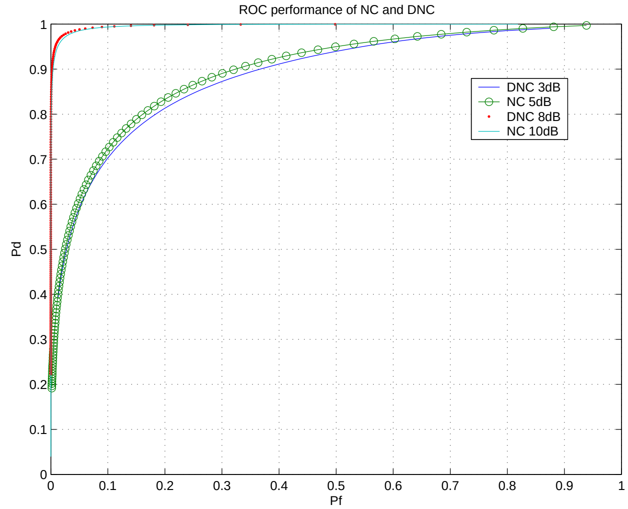


Fig. 4. Comparison of NC and DNC performance

V. Simulation Results

The DNC and NC detectors are modeled in the context of complete DSSS system in System Studio [7] to analyze their performance in a simulation environment. The test environment for complete DSSS system included random pattern generator, PN spreading, modulation and AWGN channel. The PN code used for spreading is 128-chip m-sequence. The spreading is carried out by having complete PN chip period in each information bit. BPSK chip modulation is used. The receiver is modeled with front-end filter matched to transmit pulse shaping filter. This is followed by chip matched filter and DNC (or NC) detector. The output of the detector is fed to an analyzer block which plots Probability Density Functions of both hypotheses. From these pdf plots, P_Ds and P_Fs are obtained for various values of SNR per bit and threshold values. Simulations are run with DNC and NC detectors with sufficient number of samples for statistical reliability. It is found that the simulation results match very closely with theoretical results. The Fig.(5) illustrates the simulation and theoretical results for DNC detector with γ_b=5dB.

Simulations are also conducted with non-zero carrier frequency error. It is found that the SNR degradation due to frequency error for DNC detector is same as that of NC detector. In other words, DNC enjoys the same SNR improvement, even in the presence of frequency uncertainty.

VI. Conclusion

In this paper, we have proposed a modified non-coherent detection scheme for the acquisition of DSSS signals. Analytical expressions for false alarm and detection probabilities are derived. The performance results of new scheme is compared against classical non-coherent scheme. It is shown that there is 2dB SNR advantage with the new scheme over the classical non-coherent scheme. The theoretical results are supplemented with simulation results obtained with complete modeling of the system.

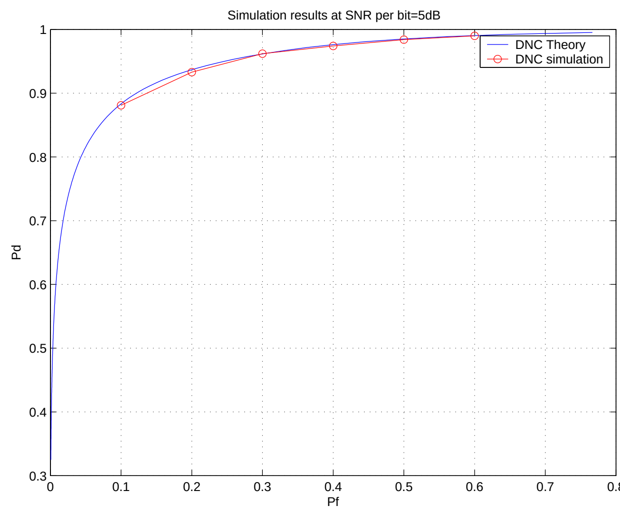


Fig. 5. Simulation results

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